Algorithmic Mechanism Design

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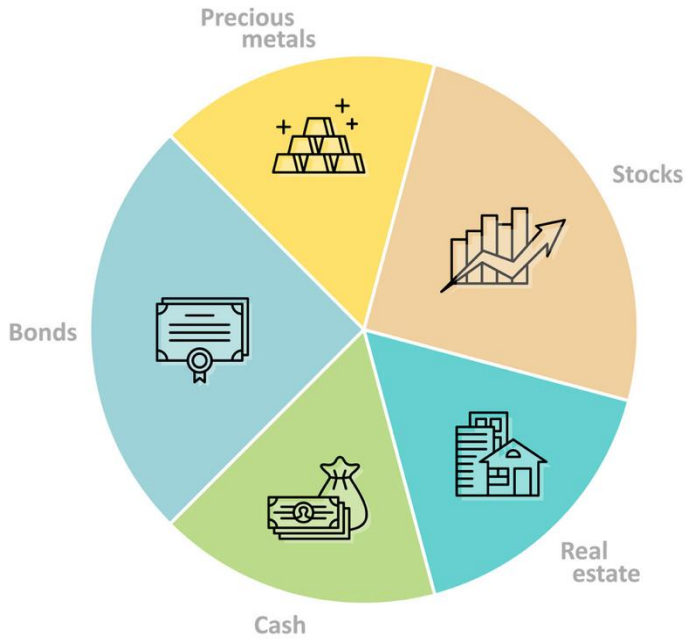
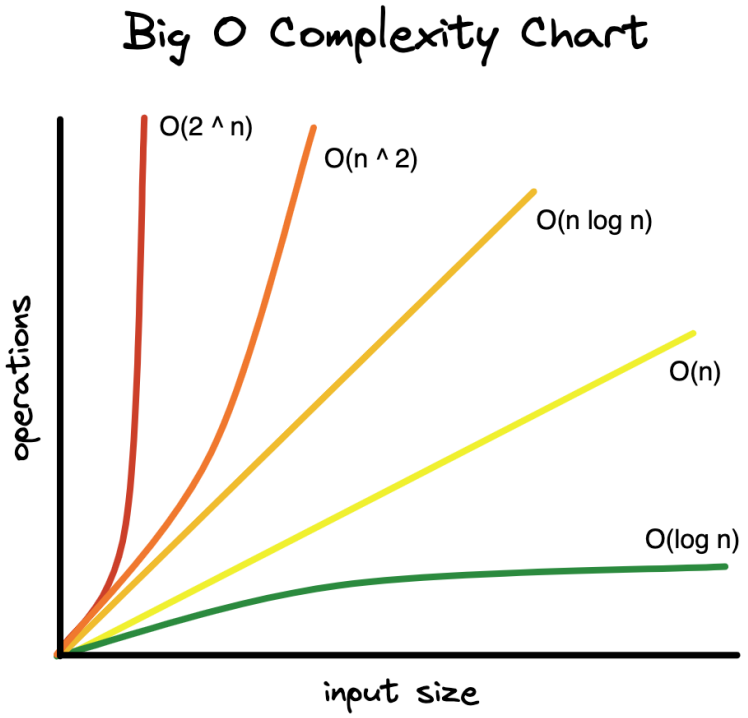
Mechanism Design

“a field in economics and game theory that explores how businesses and institutions can achieve desirable social or economic outcomes given the constraints of individuals' self-interest and incomplete information.”



Where Algorithm Comes In

Economics	Computer Science
real-world Bayesian distributions - Average Case Analysis	abstraction - Worst Case Analysis
Solution exactness	Approximations
Nil	Efficiency - computation time - memory - communication



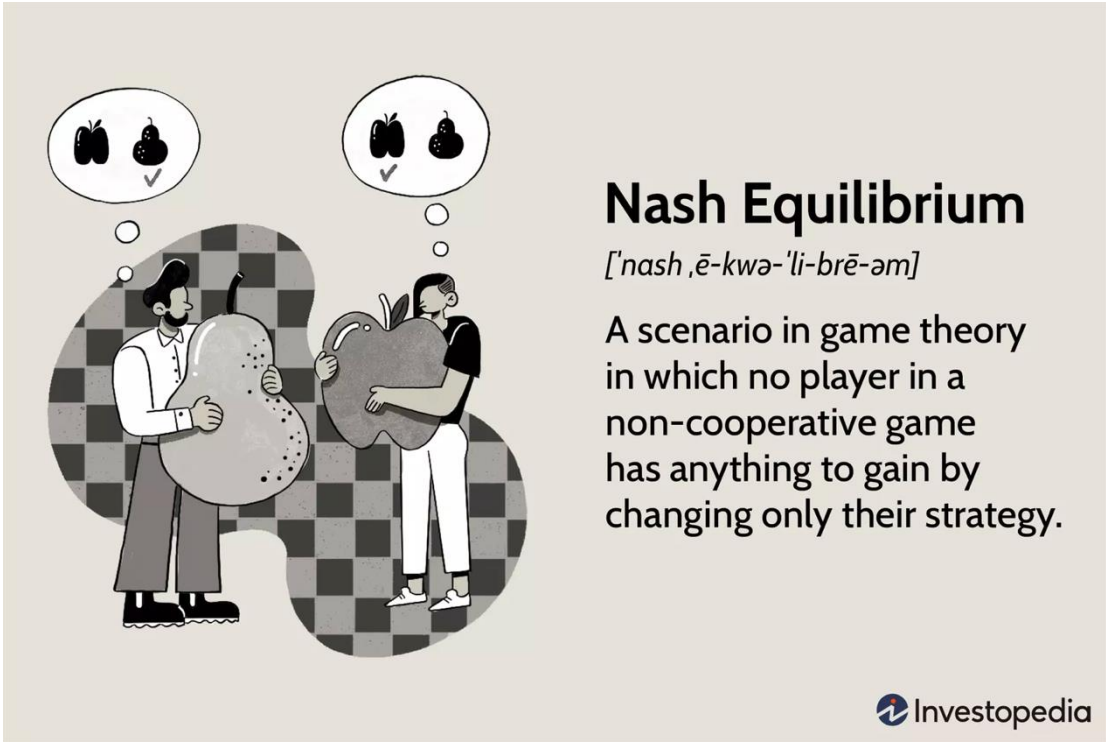
Strategic Models

- Rational Agents:
  - Utility Maximising – Strategic Moves
  - Selfishness & Lying

- Utility Model:
  - Quasi-linear utility
    - Auction: Bidder i has valuation v_i
    - utility $u_i = v_i(m_i) - p_i$
  - Value transfer

Combining Strategy and Computation

- Goal
  - Design truthful mechanisms
  - Run in polynomial time
  - Determine "optimal" social outcome



- Truthfulness:
  - aka **Incentive compatibility**
    - **Reporting false value is no better than true value**
  - Internalise **"Dominant strategy"** into the mechanism
    - Direct Revelation principle in Mechanism Design

Multi-unit Auction

Problem Definition

- Algorithmic Perspective:
  - m identical items
  - n bidders with private valuation functions (willingness to pay)
  - Give an efficient algorithm that outputs allocation $(m_1, m_2, \ldots, m_n)$
  - Maximize total social welfare $\sum v_i(m_i)$
  - Constraint $\sum m_i \leq m$

- Additional Strategic Considerations:
  - output payments $(p_1, p_2, \ldots, p_n)$
  - ensure truthfulness

UTILITY $u_i = v_i(m_i) - p_i$

Memory Considerations

- Input Representation
  - valuations for m items

    $v_i(1), v_i(2), \ldots, v_i(m)$ for $\forall i$

    -> more succinct representations?
  - single value encompasses multiple meanings

m could be very large!

Examples

- step function $v(k) = \begin{cases} 0 \text{ for } k < k^* \\ p \text{ for } k \geq k^* \end{cases}$
  - only need **(k*, p)**
- piece-wise function

$$v(\text{item } k) = \begin{cases} p_1 \text{ for } k \leq k_1 \\ p_2 \text{ for } k_1 < k \leq k_2 \\ \ldots \\ p_t \text{ for } k_{t-1} < k \leq k_t \end{cases}$$

  - **t** $\ll$ **m** pairs of (k_i, p_i)

Efficiency Requirements

- POLY time w.r.t. ?

A theory perspective:

- size of input encoding

Two data query models

1. concrete functions **(bidding language)**
   - length of all valuations
   - n, bidders
   - $\log(m)$, #bits to represent #items we bid
   - t, #bits to represent a value
     - aka precision

2. black-box communication
   - no precise input representation

   - number of "**value-queries**"

   - length of answer bits

   - still require sublinear in m!

   e.g., $\sqrt{m}, \ln m, \log m$

General Allocation Algorithm

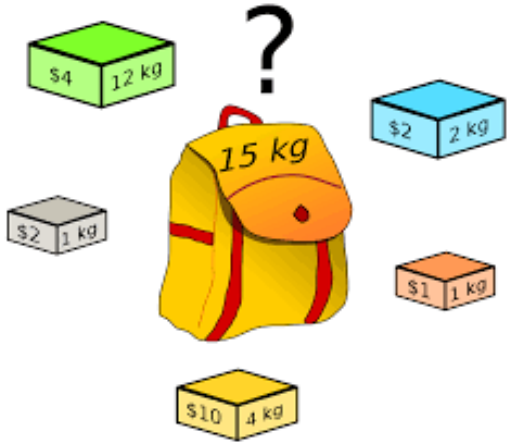
- Dynamic Programming (DP)
  - Divide-and-Conquer & Optimal Substructure
  - Subproblem:
    - Optimal allocation of first k items among the first i bidders
    - $s(i,k) = \max_{0 \le j \le k} v_i(j) + s(i-1, k-j)$

NOT computationally efficient!

$O(nm^2)$

#bidders/ items	0	1	2	…	m-1	m
0						
1						
2			$s(i,k)$			
…						
n						$s(n,m)$

Intractability



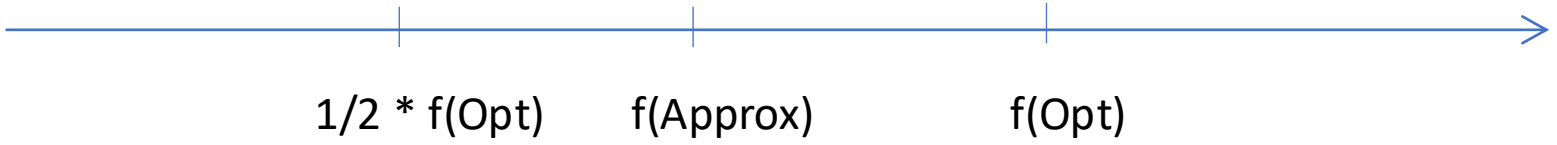
- Bidding language model
  - Consider the simplest step function $v(k) = \begin{cases} 0 \text{ for } k < k^* \\ p \text{ for } k \geq k^* \end{cases}$
  - "Knapsack problem"
    - player i = item i with value p;
    - m = knapsack capacity
    - allocate k items to player i = packing item i with weight k
    - $\sum k_i \leq m$
  - NP-complete

- Communication model $O(m)$
  - number of value queries
  - length of answer bits

Approximate Optimality

- Fully polynomial-time approximation scheme (FPTAS)
  - additional parameter $\epsilon < 1$
    - specifies how close our approximation should be
  - additional polynomial time requirement $Poly(\frac{1}{\epsilon})$
  - for maximisation problem: $f(Approx) \geq (1 - \epsilon) f(Opt)$

- α - approximation
  - for maximisation problem: $f(Approx) \geq \alpha * f(Opt)$



Truncation

- Previously, m columns in the table
  - $O(m)$ tries for each subproblem

#bidders/ total value constraint	0	1	…	…	n^2/e
0					
1	look at previous n/e cells in the row above				
…				(i, w)	
n					

- A **FPTAS** by value truncation:
  - number of possible values $w \leq n/\epsilon$
- New subproblem:
  - minimum number of items that yields total value $\geq w\delta$
- carefully choose truncation precision $\delta = \epsilon * \max v/n$
  - total relative error less than ϵ
  - $(1 - \epsilon)$ approximation

Truthful Auctions

- VCG Mechanism

- Choice of payment function
  - Align players' max utility with total social welfare

$$u_i = v_i(m_i) - p_i = v_i(m_i) + \sum_{j \neq i} v_j(m_j) - \sum_{j \neq i} v_j(m'_j)$$

$$p_i = \sum_{j \neq i} v_j(m'_j) - \sum_{j \neq i} v_j(m_j)$$

optimal allocation that maximises other players' welfare

1. no positive transfer
2. individual rationality

- Sadly,
  - $O(nm^2)$ General Allocation Algorithm

Truthful Approximation?

- Counter Example

- Idea:

allocation that "approximately" maximises total welfare

$$p_i = \sum_{j \neq i} v_j(m'_j) - \sum_{j \neq i} v_j(m_j)$$

- 2 players, 2 items; $v(1) = 1.9$ and $v(2) = 3.1$
- Truncation scheme: $p_i = \sum_{j \neq i} v_j(m'_j) - \sum_{j \neq i} v_j(m_j)$
  - $v(1) = 1$ and $v(2) = 3$
  - Optimal allocation: 2 items to single player
    - payment $= 3.1 - 0 = 3.1$; net utility $= 3.1 - 3.1 = 0$!
  - false reporting $v(1) = v(2) = 3$
  - Optimal allocation: 1 item to each
    - payment $= 3.1 - 1.9 = 1.2$; net utility $= 1.9 - 1.2 > 0$!

Not incentive compatible!

Restricted VCG

- Maximum in Range

- 2-approximation scheme
- Idea:
  - maximise over a restricted range of possible allocations
  - use General Allocation Algorithm
- bundles of items
  - manipulate m into some form in n
  - e.g., m/n^2 items in a bundle -> n^2 items in total

the only approximation mechanisms that become truthful under the VCG payment rule.

Proof

m/n^2

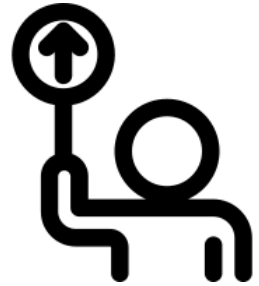
cannot be better than 2!

- bidder who gets the greatest number of items
  - Case 1: at least half total value from him

    -> give everything to him

  - Case 2: less than half from him

    -> take everything from him; distribute to all others



$\geq \frac{1}{2} \sum v_i(m_i)$

- our allocation always ≥ half total optimal

$\geq \frac{1}{2} \sum v_i(m_i)$

Truthful Auctions

- non- VCG Mechanism (Single Parameter)

- Two properties for Truthfulness
  - Monotonicity
    - win with bid (k_i, v_i) => win with bid $(k_i' < k_i, v_i' > v_i)$
  - Critical payment
    - the minimum value for a win with k items

$$v(k) = \begin{cases} 0 \text{ for } k < k^* \\ p \text{ for } k \geq k^* \end{cases}$$

- Idea:
  - still use δ scale truncation to keep range of values small -> efficiency
  - monotone way of choosing δ -> truthfulness
    - simultaneously try multiple values of δ
    - independent of bids $(k*, m)$-> but trim down search space to $\frac{2n^2}{\epsilon}$
- **FPTAS!**

Beyond

- Randomisation
  - maximise **expected** social welfare
  - maximum-in-distributional-range
  - FPTAS
    - reduce the number of values to consider for each player
- Combinatorial Auctions
  - m heterogenous items
  - allocation of a subset of items
  - valuation of all subsets

References

Nisan, Noam. (2015). Algorithmic Mechanism Design: Through the lens of Multiunit auctions. Handbook of Game Theory with Economic Applications. 4. 477-515. 10.1016/B978-0-444-53766-9.00009-4.